97/100

Birzeit University
Department of Mathematics
math 233

First hour exam
Name: ISSRA AL-ZURBA

Fall 2014
Number: 1121649

Question# 1 (18%) Consider the following statement "ab is divisible by 4 whenever both a and b are even "

i)write this statement in the form " if p then q"

if a and b are both even then ab is divisible by 4
if a and b are both-even then 4 divides ab

4 divides ab
$\Rightarrow \frac{ab}{4} = k$

ii)Write the converse of this statement

if ab is divisible by 4 then a and b are both even
if 4 divides ab then a and b are both even

iii)Write the contrapositive of this statement

if 4 doesn't divide ab then a is odd or b is odd

iv)Write the negation of this statement

a and b are both even and 4 doesn't divide ab

v)Prove this statement

proof (direct)

suppose a is even and b is even

$\Rightarrow a = 2k \quad , b = 2s \quad , s, k \in \mathbb{Z}$

$\Rightarrow ab = (2k)(2s) = 4(ks) = 4s' \quad , s' \in \mathbb{Z}$

$\Rightarrow \frac{ab}{4} = s'$

$\Rightarrow$ 4 divides ab

m divides n
$\frac{n}{m} = k$

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Question #2 (20%): Which of the following statements is true and which is false? Justify your answer

1) F If $\{x\} \in A$, and $A \in B$ then $\{x\} \in B$

counter example

$A = \{\{x\}, 1\}$

$B = \{\{\{x\}, 1\}, 2\} = \{A, 2\} \Rightarrow A \in B$

$\{x\} \in A$ but $\{x\} \notin B$

2) T $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x \le y^2)$

proof

$x = 0$

$0 \le y^2 \quad \forall y \in \mathbb{Z}$

3) F If $A \cap B = A \cap C$ then $B = C$

counter example

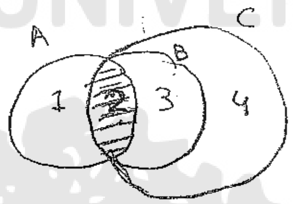
$A = \{1, 2\}$

$B = \{2, 3\}$

$C = \{2, 3, 4\}$

$A \cap B = \{2\}$, $A \cap C = \{2\}$

$A \cap B = A \cap C$ but $B \neq C$



4) T If $B \subset A$ then $A - B \neq \phi$

proof (direct)

suppose $B \subset A$ — $x \in A$ and $x \notin B$

$\Rightarrow (x \in B \to x \in A)$ and $A \neq B$

$\Rightarrow \exists\, y \in A$ and $y \notin B$

$\Rightarrow y \in (A - B) \Rightarrow A - B \neq \phi$

5) T $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})(z > x + y)$

proof

suppose $x + y = n$

and $z = n + 1$

$\Rightarrow n + 1 > n$

$\Rightarrow z > x + y$ for $(\forall x \in \mathbb{R})$ and $(\forall y \in \mathbb{R})$

Question # 3 (18%):Prove the following

1)For any sets A,B,C , $(A-C)-(B-C) \subseteq (A-B)$

Proof (direct)

$x \in (A-C)-(B-C)$

$\Rightarrow x \in [(A-C) \cap (B-C)']$

$\Rightarrow x \in (A-C)$ and $x \in (B-C)'$

$\Rightarrow x \in (A \cap C')$ and $x \in (B \cap C')'$

$\Rightarrow x \in (A \cap C')$ and $x \in (B' \cup C)$

$\Rightarrow (x \in A$ and $x \notin C)$ and $(x \notin B$ or $x \in C)$

$\Rightarrow \underline{x \in A}$, and $x \notin C$ and, $\underline{x \notin B}$ or $x \in A$ and $(x \notin C$ and $x \in C)$

$\Rightarrow x \in A$ and $x \notin B$ or $x \in A$ and $\times$

$\Rightarrow x \in (A-B)$ or $\times$ $\Rightarrow x \in (A-B)$ $\Rightarrow (A-C)-(B-C) \subseteq (A-B)$

2)For any sets A,B If $A \subseteq B$ then $\rho(A) \subseteq \rho(B)$

Proof (direct)

Suppose $A \subseteq B$ and let $\{x\} \in P(A)$

$\Rightarrow x \in A$

$A \subseteq B$ $\Rightarrow x \in B$

$\Rightarrow \{x\} \in P(B)$

$\Rightarrow P(A) \subseteq P(B)$

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Proof (direct)

Suppose $A \subseteq B$ and let $C \in P(A)$

$\Rightarrow C \subseteq A$

$\Rightarrow C \subseteq B$

$\Rightarrow C \in P(B)$

$\Rightarrow P(A) \subseteq P(B)$

3)If a, b are reals and $0<a<1<b$ then $a+b>1+ab$

Proof (contradiction)

Suppose $0<a<1<b$ and $a+b \leq 1+ab$

$\Rightarrow a+b-ab-1 \leq 0$

$\Rightarrow a(1-b)+b-1 \leq 0$

$\Rightarrow a(1-b)-(1-b) \leq 0$

$\Rightarrow (1-b)(a-1) \leq 0$

$\Rightarrow (1-b) \geq 0$ and $(a-1) \leq 0$ or $(1-b) \leq 0$ and $(a-1) \geq 0$

$\Rightarrow b \leq 1$ and $a \leq 1$ or $b \geq 1$ and $a \geq 1$

$\Rightarrow \times$ and $\times$ or $\times$ and $\times$

$\Rightarrow \times$ or $\times$

$\Rightarrow \times$

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Question # 4 (10%):Let A={{2},5} , B={5 , 6 , 2} , C={5 , 2 , 7 } , find

i) $A \cap B$ = {5}

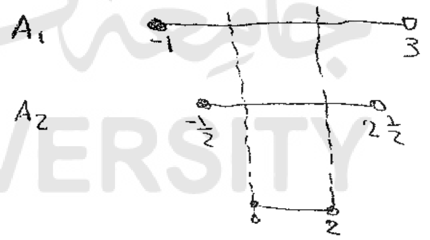
ii) A − C ·

A-C = {{2}}

iii) $\rho(A)$ = { ϕ , {{2}} , {5} , {{2},5} }

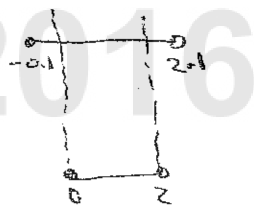
Question #5 (16%) for each $k \in \mathbb{N}$ let

$A_k = [-\frac{1}{k}, 2+\frac{1}{k})$, find



a) $\bigcup_{k=1}^{\infty} A_k$ = [-1, 3)

b) $\bigcap_{k=1}^{\infty} A_k$ = [0, 2]



$\bigcup_{k=10}^{\infty}$ = [-0.1, 2.1)

c) $\overline{(\bigcup_{k=10}^{\infty} A_k)}$ = $\mathbb{R}$ − [-0.1, 2.1)

d) $\bigcap_{k=1}^{\infty} \overline{A}_k$ = $\overline{\bigcup_{k=1}^{\infty} A_k}$ = $\mathbb{R}$ − [-1, 3)

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Question # 6 (18%) prove that for each positive naturalm number n ≥ 4 , $2^n < n!$

<u>proof</u> by first extended principle of mathmatical induction

① it is true for $n=4$ <u>since</u> $2^4 < 4! \Rightarrow 16 < 24$

② suppose it is true for $n = k$

<u>i.e</u> $2^k < k!$

③ want to prove it is true for $n=k+1$

<u>i.e</u> $2^{k+1} < (k+1)!$

<u>now</u>

$$2^{k+1} = 2^k (2)^1$$
$$< k! (2)$$
$$< k! (k+1)$$

$k > 4 \Rightarrow k+1 > 5$

because $2 < k+1$

$$2^{k+1} < (k+1)!$$

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$\frac{94+2}{100}$

Birzeit University
Department of Mathematics
Math 243

Second hour exam
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Section#...12830 – 14800 ...

Spring 2014
Number:..1121649

Questio#1(30%) Prove or disprove each of the following statements

a)If $A \neq \emptyset$ and $A\times B = A\times C$ then $B=C$ True

Proof suppose $A\times B = A\times C$ and

$\boxed{\subseteq}$ let $y \in B \Rightarrow (x,y) \in A\times B$ (since $A \neq \emptyset$)

$\Rightarrow (x,y) \in A\times C$ (Since $A\times B = A\times C$)

$\Rightarrow y \in C$

$\Rightarrow B \subseteq C$

$\boxed{\supseteq}$ let $y \in C \Rightarrow (x,y) \in A\times C$ (since $A \neq \emptyset$)

$\Rightarrow (x,y) \in A\times B$ (Since $A\times C = A\times B$)

$\Rightarrow y \in B \Rightarrow C \subseteq B \Rightarrow B = C$

b)If f and g are functions then $f \cup g$ is a function False

counterexample

$f = \{(1,2)\} \rightarrow$ function

$g = \{(1,3)\} \rightarrow$ function

$f \cup g = \{(1,2), (1,3)\}$ not function

c)If R and S are equivalence relations on A then $R \cup S$ is an equivalence relations on A

False

not transitive

Counter example

$R = \{(4,6), (2,3)\}$ tran.

$S = \{(1,2), (3,4)\}$ tran.

$R\cup S = \{(4,6), (2,3), (1,2), (3,4)\}$

not tran. Since $(2,4) \notin R\cup S$

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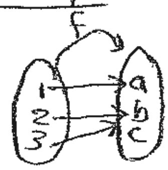
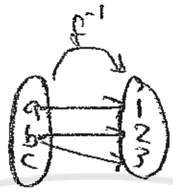
d) If f is a function then f^{-1} is a function   False

counter example



function



not function

e) If R,S are transitive then $S \circ R$ is transitive

False

Counterexample

$R = \{(1,2),(3,4)\}$ tran.

$S = \{(4,6),(2,3)\}$ tran.

$S \circ R = \{(1,3),(3,6)\}$ not transitive since $(1,6) \notin S \circ R$

f) If R and S are symmetric relations then $(R \circ S)^{-1} = S \circ R$

~~False~~

~~Counter example~~

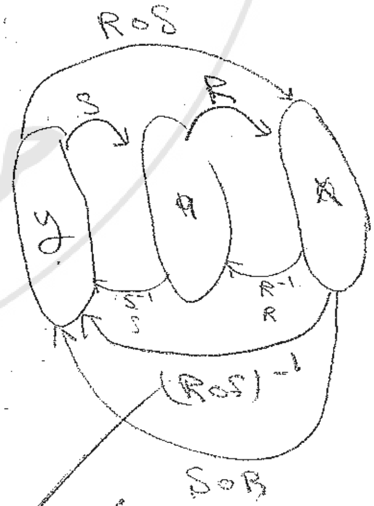
True

Proof

$(x,y) \in (R \circ S)^{-1} \Leftrightarrow \exists a,\ (x,a) \in R^{-1}$ and $(a,y) \in S^{-1}$

$\Leftrightarrow (x,a) \in R$ and $(a,y) \in S$

$\Leftrightarrow (x,y) \in S \circ R$



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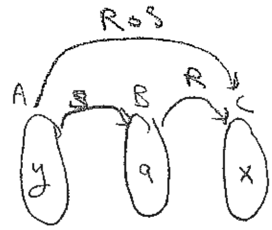
Question #2(20%) a)Prove thatif A,B,C be sets, and let $R \subseteq B\times C, S \subseteq A\times B$ be relations then $(R\circ S)^{-1} = S^{-1}\circ R^{-1}$

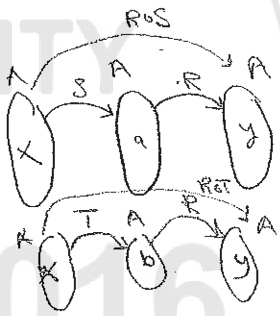


proof $(x,y) \in (R\circ S)^{-1} \Leftrightarrow (y,x) \in R\circ S$

$\Leftrightarrow \exists a \in B;\ (y,a) \in S$ and $(a,x) \in R$

$\Leftrightarrow \exists a \in A;\ (a,y) \in S^{-1}$ and $(x,a) \in R^{-1}$

$\Leftrightarrow (x,y) \in S^{-1}\circ R^{-1}$



b)Prove tha if R,S,T be relations from A to A then $(R\circ S)\cup(R\circ T) \subseteq R\circ(S\cup T)$

$R,S,T \subseteq A\times A$

proof

let $(x,y) \in (R\circ S)\cup(R\circ T) \Rightarrow (x,y) \in R\circ S$ or $(x,y) \in R\circ T$

$\Rightarrow \exists a;\ (x,a) \in S$ and $(a,y) \in R$

or $\exists b;\ (x,b) \in T$ and $(b,y) \in R$

~~⇒ (x,a)∈S and (x,b)∈T or (x,a)∈S and (b,y)∈R or (a,y)∈R and (x,b)∈T or (a,y)∈R and (b,y)∈R~~

$\Rightarrow ((x,b) \in S$ or $(x,b) \in T)$ and $(b,y) \in R$

$\Rightarrow (x,b) \in (S\cup T)$ and $(b,y) \in R$

$\Rightarrow (x,y) \in R\circ(S\cup T)$

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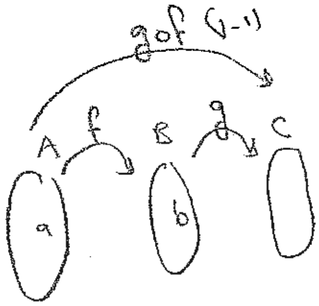
Question #3(20%) Let A, B, C be nonempty set and let $f: A \to B$, $g: B \to C$ be functions.

a) Show that if $g \circ f$ is one to one, then $f: A \to B$ is one to one.



f (1-1)??

proof suppose $f(a) = f(b)$

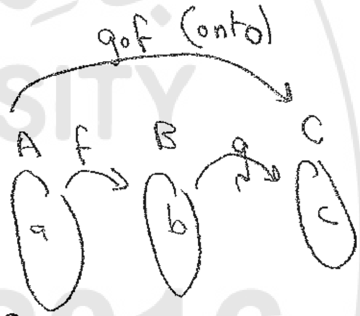
$\Rightarrow g(f(a)) = g(f(b))$

$\Rightarrow g \circ f(a) = g \circ f(b)$

$\Rightarrow$ ~~[illegible]~~ $a = b$ Since $g \circ f$ is (1-1)

$\Rightarrow f$ is one to one

b) Show that if $g \circ f$ is onto, then $g: B \to C$ is onto.



g onto??

proof let $c \in C \Rightarrow \exists a \in A$; $g \circ f(a) = c$ (onto)

$\Rightarrow (a, c) \in g \circ f$

$\Rightarrow \exists b \in B$; $(a, b) \in f$ and $(b, c) \in g$

$\Rightarrow \exists b \in B$; $(b, c) \in g$

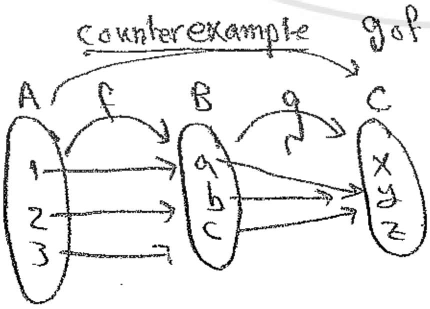
$\Rightarrow g(b) = c$

g is onto

c) Show that the converse of (a) is not true

converse : If $f: A \to B$ is one to one, then $g \circ f$ is one to one.

False counterexample



f is one to one but $g \circ f$ is not one to one

Question #4(20%)

Let $X=\mathbb{R}\times\mathbb{R}$, For each real number b let $D_b=\{(x,y)\in X: y=x+b\}$

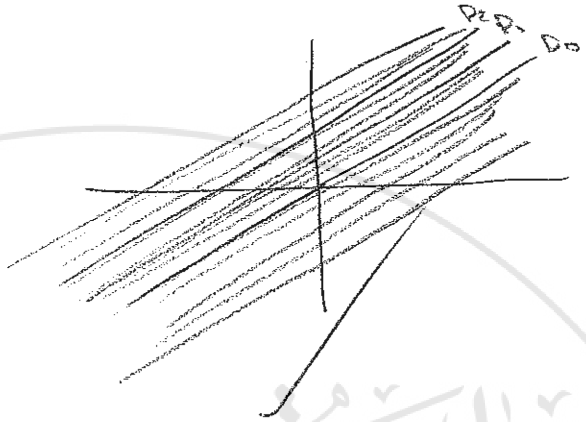
a) Is $\{D_b: b\in\mathbb{R}\}$ a partion of X? Prove your answer?

$D_1=\{(x,y)\in X : y=x+1\}$

$D_0=$ " $y=x$

$D_{-1}=$ " $y=x-1$

$\vdots$



هنا الرسم

1) $\phi\notin D_b$

2) لا يوجد تقاطع بين خط و خط

Since they are parallel and the slope $=1$ for all $b\in\mathbb{R}$

3) $\bigcup_{b\in\mathbb{R}} D_B=\mathbb{R}\times\mathbb{R}$ اكان الخطوط كلها يعطي $\mathbb{R}\times\mathbb{R}$

$\Longrightarrow D_b$ is apartion of X

b)Define a relation R on X by (s,t)R(u,v) if and only if there is a real number b such that (s,t)and (u,v) both belongs to D_b, for some $b\in\mathbb{R}$

Is R Reflexive ? Symmetric? Transitive? Explane your answer

$(s,t)\,R\,(u,v)$ iff $t-s=b=v-u \Rightarrow t-s=v-u$

ref $(x,y)\,R\,(x,y)$ since $y-x=y-x$ ✓ ref ✓

symm $(a,b)\,R\,(c,d) \Rightarrow b-a=d-c$

$\Rightarrow d-c=b-a$

$\Rightarrow (c,d)\,R\,(a,b)$ ✓ symm ✓

tran let $(a,b)\,R\,(c,d)$ and $(c,d)\,R\,(e,f)$

$\Rightarrow b-a=d-c$ and $d-c=f-e$

$\Rightarrow b-a=f-e$

$\Rightarrow (a,b)\,R\,(e,f)$ ✓ tran. ✓

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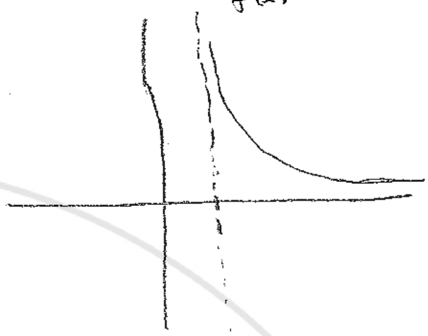
Question #5(10%) a) Let $f(x)=\dfrac{1}{\sqrt{x-1}}$, $g(x)=\sqrt{4-x}$. Find the domain of f , g , f ∘ g

* $f(x)=\dfrac{1}{\sqrt{x-1}}$

domain $x-1>0 \Rightarrow x>1$

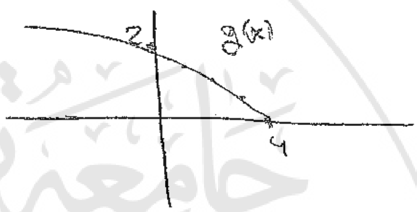
range $(0,\infty)$



* $g(x)=\sqrt{4-x}$

domain $4-x\geq 0 \Rightarrow x\leq 4$

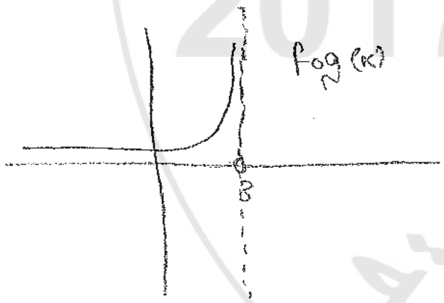
range $[0,\infty)$



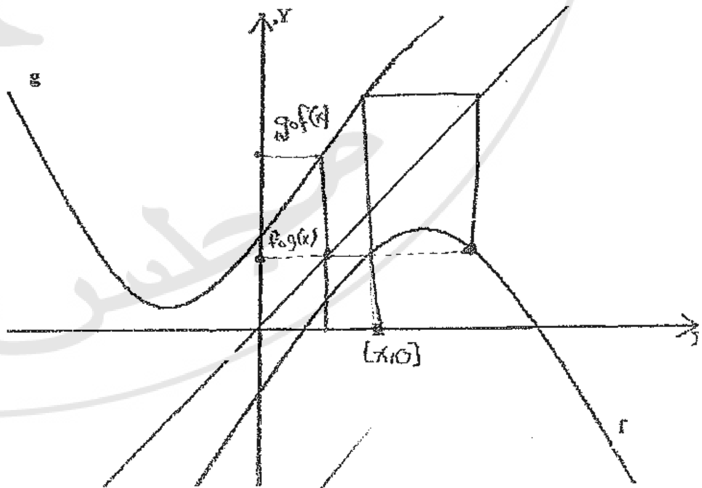
* $f\circ g = f(g(x)) = f(\sqrt{4-x}) = \dfrac{1}{\sqrt{\sqrt{4-x}-1}}$

domain $\sqrt{4-x}-1>0 \Rightarrow \sqrt{4-x}>1 \Rightarrow 4-x>1 \Rightarrow x<3$

range $(0,\infty)$



b)Use the graph of the functions f , and g
to indicate on the graph
the value of (f ∘ g) (x) and (g ∘ f)(x)



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